

Simultaneous Conduction and Radiation Energy Transfer

Radiant energy can transfer from a colder to a warmer radiator.

#####, PhD Chemical Process Control Systems Engineer, PE TX & CA

Abstract

The rigorous model of simultaneous thermal and radiant energy transfer proves energy transfer by radiation can flow from a colder radiator to a warmer one, heating the warmer one further. It explains and quantifies how dissimilar walls in a room can have different steady-state temperatures. Only the general laws of thermal and radiant energy transfer and the First Law of Thermodynamics, conservation of energy, are employed.

Introduction

Many claim radiant energy can only transfer from a hot radiator to a colder one. Otherwise the Second Law of Thermodynamics would be violated. While this is true for thermal energy transfer by conduction or convection, it is not true for radiant energy transfer.

How does a cold radiator transfer energy to warmer surroundings? It depends on absorptivity, emissivity and intensity differences at each radiating surface and on the presence of simultaneous conduction. We will show why temperatures of dissimilar room walls at steady-state are not be equal.

Conservation of Energy

Consider an object radiator at temperature T in a room of air and radiating surroundings at temperature T_s .

The First Law of Thermodynamics, conservation of energy for the radiator says: Rate of energy out/in by conduction at T to/from surroundings at T_s = rate of energy in/out by radiation from/to surroundings minus/plus the rate of energy accumulation/depletion within the radiating body.

Rate Out = Rate In - Rate of Accumulation

$$Q_o(t) = Q_i(t) - m C_p dT(t)/dt$$

Q 's are energy transfer rates, m is body mass and C_p is its heat capacity. This ordinary differential equation can be integrated for transient response $T(t)$ for given $Q_i(t)$ and $Q_o(t)$ functions of T , T_s and time, t , even when Q 's depend on T .

At steady state, T is constant, $dT/dt = 0$, out/in = in/out and

$$Q_o = Q_i \tag{1}$$

Let Q_c be rate by conduction and Q_r rate by radiation. Q_o is Q_c if $Q_c > 0$. Q_i is Q_r if $Q_r > 0$.

$$Q_c = Q_r$$

According to the First Law each Q must have the same sign, in = out. By convention, if both are positive, thermal energy flows out, radiant energy flows in and radiator $T >$ surroundings T_s . Radiant energy transfers from cold surroundings to warm radiator. If both are negative, thermal energy flows in, radiant energy flows out and radiator $T <$ surroundings T_s . Radiant energy transfers from cold radiator to warm surroundings. Radiant energy flow balances thermal energy flow.

This is a common source of confusion. Like all flows, energy flow has a direction, mathematically it is a vector. One must use care to get signs and directions right.

Energy Transfer Rate Laws

The general law of thermal energy transfer from a body at T to its surroundings at T_s by conduction and convection is

$$Q_c = k(T - T_s), \text{ w/m}^2 - \text{K}/100 \quad (2)$$

where $k > 0$ is the of thermal energy transfer coefficient. It depends on materials and mode of conduction/convection. Assume $k = 100 \text{ w/m}^2 - \text{K}/100$. Define T and $T_s = \text{deg K}/100$.

If $Q_c > 0$, flow is Q_o from body out to the surroundings; if $Q_c < 0$, flow is Q_i in to body from the surroundings.

The general law of radiant energy transfer¹ from surroundings to a radiator, derived in Appendix is:

$$Q_r = 5.67 * (\alpha \epsilon_s T_s^4 - \alpha_s \epsilon T^4), \quad (3)$$

where α and ϵ are the absorptivity and emissivity of the radiator and α_s and ϵ_s are the absorptivity and emissivity of the surroundings. Each of these properties depends on composition, temperature and pressure. (I will assume they are constant here without loss of generality.) Equation (3) was derived¹ from Stefan – Boltzmann Radiation Law.

If $Q_r > 0$, flow is Q_i in from surroundings to the radiator; if $Q_r < 0$, flow is Q_o out to surroundings. Some analysts may think if $T > T_s$, $Q_r > 0$, but this is not true here because $Q_r > 0$ is defined to be Q_i .

When $T > T_s$, Q_c and $Q_r > 0$, radiant energy flows in from lower surroundings T_s to higher radiator T . The direction of radiant energy flow depends on intensity differences, not temperature differences.

When $T < T_s$, Q_c and $Q_r < 0$, radiant energy flows out from lower radiator T to higher surroundings T_s .

$$Q_c = Q_r; k(T - T_s) = 5.67 * (\alpha \epsilon_s T_s^4 - \alpha_s \epsilon T^4), \quad (4)$$

Rearranging,

$$kT + 5.67 \alpha_s \epsilon T^4 = kT_s + 5.67 \alpha \epsilon_s T_s^4; \text{LHS} = \text{RHS} \quad (5)$$

Given T_s , we can find T , or vice versa.

$T = T_s$, if and only if $\alpha \epsilon_s = \alpha_s \epsilon$, ($Q_c = Q_r = 0$), which is rarely the case for dissimilar radiators.

If $\alpha_s \epsilon > \alpha \epsilon_s$, $T < T_s$ by inspection. If $\alpha_s \epsilon < \alpha \epsilon_s$, $T > T_s$.

There will always be a solution T for any given $T_s > 0$.

When both are black bodies, $\alpha = \epsilon_s = \alpha_s = \epsilon = 1$.

$$kT + 5.67 T^4 = kT_s + 5.67 T_s^4, T = T_s \text{ by inspection.}$$

If no other energy transfer is involved and Kirchhoff Law applies to both, $\alpha = \epsilon$, $\alpha_s = \epsilon_s$

$$kT + 5.67 \epsilon_s \epsilon T^4 = kT_0 + 5.67 \epsilon_s \epsilon T_s^4, T = T_s \text{ by inspection.}$$

Poor Radiator Example

Assume $\alpha_s = 0.9$, $\epsilon_s = \mathbf{0.8}$, $\alpha = 0.6$, $\epsilon = 0.5$. $\alpha_s \epsilon = 0.45 < \alpha \epsilon_s = 0.48$.

$$100 T + 5.67 * 0.9 * 0.5 T^4 = 100 T_s + 5.67 * 0.6 * \mathbf{0.8} T_s^4$$

$$100 T + 5.67 * 0.45 T^4 = 100 T_s + 5.67 * \mathbf{0.48} T_s^4$$

Let $T_s = 20^\circ\text{C}$. We find $T = \text{about } 23.473^\circ\text{C} > T_s$

$$\text{RHS} = 100 * 2.9315 + 5.67 * \mathbf{0.48} * 2.9315^4 = 293.15 + 5.67 * \mathbf{0.48} * 73.8515$$

$$= 293.15 + \mathbf{200.9944} = 494.144369$$

$$\text{LHS} = 100 * 2.96623 + 5.67 * 0.45 * 2.96623^4 = 296.623 + 5.67 * 0.45 * 77.4139$$

$$= 296.623 + 197.5217 = 494.144356, \text{ checks}$$

$$Q_c = k(T - T_s) = 100 * (2.9662 - 2.9315) = 3.473 \text{ out}$$

$$Q_r = 5.67 * (\alpha \epsilon_s T_s^4 - \alpha_s \epsilon T^4) = 5.67 (0.6 * \mathbf{0.8} * 2.9315^4 - 0.9 * 0.5 * 2.9662^4)$$

$$= 5.67 * (\mathbf{0.48} * 73.8515 - 0.45 * 77.4139) = 5.67 * (\mathbf{35.4487} - 34.8362) = 5.67 * 0.6125 = 3.4729$$

in. Close to Q_c , round-off.

Note radiator $T = 23.47 > \text{surroundings } T_s = 20$, but colder surroundings transfer radiant energy into warmer radiator, heating it because $\alpha \epsilon_s > \alpha_s \epsilon$, $0.6 * 0.8 > 0.9 * 0.5$. $Q_r = Q_c = 3.473$.

If $T = T_s = 20$, $Q_c = 0$ and

$$Q_r = 5.67 * (\alpha_{\epsilon s} T_s^4 - \alpha_{\epsilon} T^4) = 5.67 (0.6 * \mathbf{0.8} * 2.9315^4 - 0.9 * 0.5 * 2.9315^4) = 5.67 * 2.9315^4 * (0.48 - 0.45) = 5.67 * 73.8515 * 0.03 = 12.562.$$

This means radiant energy is coming in but no thermal energy is leaving. The radiator $T = 20$ will increase, thermal energy starts to go out and incoming radiant energy decreases until a $T = 23.473$ is reached. Q_r in drops from 12.562 to 3.473 and Q_c out increases from 0 to 3.473, constant.

At steady state, radiator is $T = 23.473$ and surroundings $T_s = 20$. Energy is transferring out by conduction because $T > T_s$. Energy is transferring in by radiation because intensity of colder surroundings $>$ radiator surface.

$$Q_r = 5.67 (0.6 * \mathbf{0.8} * 2.9315^4 - 0.9 * 0.5 * 2.96623^4) = 5.67 (0.48 * 73.8515 - 0.45 * 77.4140^4) = 5.67 * (\mathbf{35.4487} - 34.8362) = 5.67 * 0.6125 = 3.473.$$

Radiant energy transfers in from surroundings at 3.473 even though surroundings $T_s = 20$ are colder than radiator at $T = 23.473$.

Since radiation rate increases as power of 4 and conduction rate increases linearly, there will always be a solution where Q_r drops and/or Q_c increases until they meet, $Q_c = Q_r$ not = 0. T_s remains 20C and radiator warms to 23.473 at steady-state.

Good Radiator Example

Exchange α_{ϵ} for $\alpha_{\epsilon s}$. Radiator has higher α and ϵ ; it is a better radiator.

Assume $\alpha_s = 0.6$, $\epsilon_s = 0.5$, $\alpha = 0.9$, $\epsilon = 0.8$. $\alpha_{\epsilon s} = 0.48 > \alpha_{\epsilon} = 0.45$.

$$100 T + 5.67 * 0.6 * 0.8 T^4 = 100 T_s + 5.67 * 0.9 * 0.5 T_s^4$$

$$100 T + 5.67 * 0.48 T^4 = 100 T_s + 5.67 * 0.45 T_s^4$$

Let $T_s = 20$ C. We find $T =$ about 16.6004C $> T_s$

$$\text{RHS} = 100 * 2.9315 + 5.67 * 0.45 * 2.9315^4 = 293.15 + 5.67 * 0.45 * 73.8515$$

$$= 293.15 + \mathbf{188.4322} = 481.582221$$

$$\text{LHS} = 100 * 2.897504 + 5.67 * 0.48 * 2.897504^4 = 289.7504 + 5.67 * 0.48 * 70.4849$$

$$= 289.7504 + 191.8317 = 481.582143, \text{ checks}$$

$$Q_c = k(T - T_s) = 100 * (2.8975 - 2.9315) = - 3.3996 \text{ out} = 3.400 \text{ in}$$

$$Q_r = 5.67 * (\alpha_{\epsilon s} T_s^4 - \alpha_{\epsilon} T^4) = 5.67 (0.9 * \mathbf{0.5} * 2.9315^4 - 0.6 * 0.8 * 2.8975^4)$$

$$= 5.67 * (\mathbf{0.45} * 73.8515 - 0.48 * 70.4849) = 5.67 * (\mathbf{33.2331} - 33.832759) = 5.67 * (- 0.5995) = - 3.3995 \text{ in or } 3.400 \text{ out. Close to } Q_c, \text{ round-off.}$$

Note radiator $T = 16.60 <$ surroundings $T_s = 20$, but colder radiator transfers radiant energy out to warmer surroundings, because $\alpha_{\epsilon} > \alpha_{\epsilon s}$, $0.6 * 0.8 > 0.9 * 0.5$. $Q_r = Q_c = - 3.400$. Since it is a good radiator, it is cooler than the poorer one. Thermal energy comes in; radiant energy goes out.

Decrease ϵ_s case.

Surroundings have lower emissivity.

Assume $\alpha_s = 0.9$, $\epsilon_s = 0.7$, $\alpha = 0.6$, $\epsilon = 0.5$. $\alpha_s \epsilon = 0.45 > \alpha \epsilon_s = 0.42$.

$$100 T + 5.67 * 0.9 * 0.5 T^4 = 100 T_s + 5.67 * 0.6 * 0.7 T_s^4$$

$$100 T + 5.67 * 0.45 T^4 = 100 T_s + 5.67 * 0.42 T_s^4$$

Let $T_s = 20^\circ\text{C}$. We find $T = \text{about } 16.436^\circ\text{C} < T_s$

$$\text{RHS} = 100 * 2.9315 + 5.67 * 0.42 * 2.9315^4 = 293.15 + 5.67 * 0.42 * 73.8515$$

$$= 293.15 + 175.8701 = 469.020073$$

$$\text{LHS} = 100 * 2.8959 + 5.67 * 0.45 * 2.8959^4 = 289.586 + 5.67 * 0.45 * 70.3250$$

$$= 289.586 + 179.4342 = 469.020099, \text{ checks}$$

$$Q_c = k(T - T_s) = 100 * (2.8959 - 2.9315) = - 3.565 \text{ out or } +3.565 \text{ in}$$

$$Q_r = 5.67 * (\alpha \epsilon_s T_s^4 - \alpha_s \epsilon T^4) = 5.67 * (0.6 * 0.7 * 2.9315^4 - 0.9 * 0.5 * 2.8959^4)$$

$$= 5.67 * (0.42 * 73.8515 - 0.45 * 70.3250) = 5.67 * (31.0176 - 31.6462) = - 5.67 * 0.6286 = - 3.564, \text{ in or } +3.564 \text{ out. Close to } Q_c, \text{ round-off.}$$

The minus signs mean heat transfers out by conduction and in by radiation, at same rates.

Note radiator $T = 16.436 < \text{surroundings } T_s = 20$, so conduction is from warm surroundings to cold radiator. But cold radiator, $T = 16.436$, transfers radiant energy to warmer surroundings, $T_s = 20$, because $\alpha \epsilon_s < \alpha_s \epsilon$, $0.6 * 0.7 < 0.9 * 0.5$. $Q_r = Q_c = - 3.564$. Since surroundings emit with lower intensity, rates drop and radiator cools.

If $T = T_s = 20$, $Q_c = 0$ and

$$Q_r = 5.67 * (\alpha \epsilon_s T_s^4 - \alpha_s \epsilon T^4) = 5.67 * (0.6 * 0.7 * 2.9315^4 - 0.9 * 0.5 * 2.9315^4)$$

$$= 5.67 * (0.42 - 0.45) * 2.9315^4 = 5.67 * (- 0.03) * 73.8515 = - 5.67 * 0.6286 = - 12.562 \text{ in or } 12.562 \text{ out. This is an unsteady-state.}$$

This means radiant energy is going out but no thermal energy is coming in. This is not a steady-state situation. The radiator $T = 20$ will decrease, thermal energy starts to go out and incoming radiant energy decreases until a $T = 16.436$ is reached. Q_r in drops from $- 12.562$ to $- 3.564$ and Q_c out increases from 0 to $- 3.565$, constant.

At steady state, radiator is $T = 16.436$ and surroundings $T_s = 20$. Energy is transferring in by conduction because $T < T_s$. Energy is transferring out by radiation because intensity of warmer surroundings $<$ radiator surface.

$$Q_r = 5.67 (0.6 * 0.7 * 2.9315^4 - 0.9 * 0.5 * 2.8959^4) = 5.67 * (0.42 * 73.8515 - 0.45 * 70.3251) = 5.67 * (31.0176 - 31.6463) = 5.67 * (- 0.6286) = -3.564.$$

Radiant energy transfers out to surroundings at -3.564 even though surroundings $T_s = 20$ are warmer than radiator at $T = 16.436$.

Since radiation rate increases as power of 4, as conduction rate increases linearly, there will always be a solution where Q_r increases and Q_c decreases until they meet, $Q_c = Q_r = -3.564$, not $= 0$. T_s remains 20°C and radiator cools to 16.436 at equilibrium.

With simultaneous energy transfer by conduction and radiation between a body and its surroundings at steady-state, radiant energy always transfers from the colder to the warmer because energy transfers by conduction from the warmer to the colder. This is true in the atmosphere at Earth's surface.

Confusion

Many err claiming radiant energy cannot transfer from the colder radiator to the warmer one, heating it further, because they incorrectly assume the driving force (at a distance) is temperature difference, which is true for conduction/convection through a matter field, while the driving forces for radiant energy transfer between radiators are intensity differences at radiator surfaces through a radiation field. As proved by Martin Hertzberg¹⁵.

Radiant energy does not transfer due to a temperature difference at a distance. Temperature is a point property of matter proportional to the kinetic energy of its atoms and molecules. The transfer directions switch at $T = T_s$, when $Q_c = Q_r$. Radiation direction is always opposite to conduction direction according to the First Law.

So when surroundings are T_s , a room wall at T can be greater or less than T_s , depending on its radiating properties compared to radiating properties of surroundings. Rate in by conduction = rate out by radiation = a nonzero constant at different temperatures, unless Kirchhoff's Law applies. When it does the temperatures are equal and no energy transfers between the radiator and surroundings either way.

Atmosphere

Since atmospheric temperature decreases with altitude, why doesn't energy transfer up by conduction, equalizing T above? Because thermal energy indicated by T , is kinetic energy of molecular motion and it must decrease as potential energy increases with altitude in Earth's gravitational field to maintain fixed total energy of each m^3 of gas. Another energy mechanism is involved.

Some popular explanations of Green House Gas Theory say radiant energy transfers from cold atmospheric CO_2 down to warmer surface, which absorbs it, warming it further, i. e. global warming. Actually surface partly radiates directly to space because atmosphere has some transmissivity and the rest is absorbed by the atmosphere, including trace $400 \text{ ppm } \text{CO}_2$, and then reemitted to space.

A recent paper² derived rigorous equations for the coupled atmosphere and surface temperatures. Only system properties are needed, no empiricism. When one warming and three cooling mechanisms are included in the whole system, the net effect of CO₂ on temperature is small and likely < 0.

The remaining question to quantify the effect of CO₂ changes on temperatures is: how much does CO₂ affect the atmosphere's radiating properties: absorptivity and emissivity? Assuming² a 1% increase in the atmosphere's radiating properties, perhaps due to increased CO₂, surface temperature change is - 0.76C and atmosphere change is -0.39C. The net effect is slight cooling.

Applying (3) we see $Q_r > 0$ for transfer from surface up to and absorbed by atmosphere, even when $T_s > T_a$. That is because there is no conduction involved, as proved above. The presence of radiating CO₂ increases atmosphere emissivity, a resistance to radiant energy transfer. CO₂ is not an energy blocker or trapper; it is an absorber and transmitter. No radiant energy transfers from cold atmosphere with CO₂ down to warm surface, warming the surface.

A shiny white car has greater reflectivity than rough black car. So it has lower absorptivity and emissivity. Since white absorbs less radiant energy than black, it emits less, causing it to be cooler.

Increase T_s case

Surroundings warm up.

Assume $\alpha_s = 0.9$, $\epsilon_s = 0.8$, $\alpha = 0.6$, $\epsilon = 0.5$ as before.

Let $T_s = 30C$. We find $T =$ about $33.688C > T_s$

$$RHS = 100 * \mathbf{3.0315} + 5.67 * 0.48 * 3.0315^4 = \mathbf{303.15} + 5.67 * 0.48 * \mathbf{84.4560}$$

$$= 303.15 + \mathbf{229.8553} = 533.005334$$

$$LHS = 100 * \mathbf{3.06838} + 5.67 * 0.45 * \mathbf{3.06838}^4 = \mathbf{306.838} + 5.67 * 0.45 * \mathbf{88.6414}$$

$$= 306.838 + 226.1685 = 533.006515, \text{ checks}$$

$$Q_c = k(T - T_s) = 100 * (3.06838 - 3.0315) = 3.688 \text{ out}$$

$$Q_r = 5.67 * (\alpha \epsilon_s T_s^4 - \alpha_s \epsilon T^4) = 5.67 (0.6*0.8 * \mathbf{3.0315}^4 - 0.9*0.5 * \mathbf{3.06838}^4)$$

$$= 5.67 * (0.48 * \mathbf{84.4560} - 0.45 * 88.6414) = 5.67 * (\mathbf{40.5387} - \mathbf{39.8886}) = 5.67 * \mathbf{0.65023} = 3.6868 \text{ in. Close to } Q_c, \text{ round-off.}$$

Note radiator $T >$ surroundings T_s , but colder surroundings transfer radiant energy into warmer radiator, heating it because $\alpha \epsilon_s > \alpha_s \epsilon$, $0.6*0.8 > 0.9*0.5$. $T - T_s$ increases from 3.473 to 3.688 or + 0.215C when T_s increases from 20 to 30C.

Put one hand on a mirror or metal faucet and the other on a fiberboard wall. The wall is warmer because it is a better absorber/emitter. Mirror and metal are better reflectors.

Table of Solutions

α	ϵ	α_s	ϵ_s	$\alpha \epsilon_s$	$\alpha_s \epsilon$	T_s	T	$T - T_s$
----------	------------	------------	--------------	---------------------	---------------------	-------	-----	-----------

0.4	0.2	0.8	0.4	0.16	0.16	20	20.000	0.000
0.6	0.5	0.9	0.8	0.48	0.45	---	23.473	3.473
0.9	0.8	0.6	0.5	0.45	0.48	---	16.600	- 3.400
----	----	----	0.7	0.42	----	---	16.436	- 3.5641
----	----	----	0.8	0.48	----	30	33.688	3.6877

Radiation Only

In a vacuum there is no conduction, $k = 0$. $Q_c = 0 = Q_r$ and $\alpha \epsilon_s T_s^4 = \alpha_s \epsilon T^4$.

Given T_s we find $T^4 = (\alpha \epsilon_s / \alpha_s \epsilon) T_s^4$.

$$T = (\alpha \epsilon_s / \alpha_s \epsilon)^{0.25} T_s.$$

If $(\alpha \epsilon_s / \alpha_s \epsilon) > 1$, $T > T_s$.

$$\text{Let } T = (0.48 / 0.45)^{0.25} T_s = 1.016265 * T_s.$$

$$\text{If } T_s = 20, T = 1.016265 * 2.9315 = 2.97918 = 24.768^\circ\text{C}$$

This means steady-state temperature, T , of a radiator in vacuum can differ from its radiating surrounding temperature, T_s , with no energy transferring in or out. It radiates with same intensity as surroundings.

Conclusion. According to the First Law of Thermodynamics, radiant energy transfers from a cold to a warmer radiator in the presence of energy transfer by conduction the other way. The temperature difference between radiators at steady-state depends on their radiating properties: absorptivity and emissivity.

This paper is not a mere theory because it is based on well-known laws of physics and mathematics, confirmed by observation.

Appendix. Derivation of General Radiant Energy Transfer Law Between Two Radiators¹

First, we employ the basic law for intensity of all radiators, the Stefan – Boltzmann Law.

$$I, \text{ w/m}^2 = 5.67 \epsilon T^4 \tag{4}$$

where ϵ is the emissivity of the radiator and $T = K/100$ and K is radiator temperature, deg Kelvin. Like absorptivity, emissivity has a spectrum of intensity vs. wavelength unique to each atom and molecule.

Earth's surface entities are taken to be at some average temperature, $T(h)$, and to have an average emissivity of $e(h)$ and an average absorptivity of $a(h)$. The gaseous atmosphere, without clouds to begin with, is approximated as two concentric layers that are partially absorbing,

partially transparent, and non-reflective. The lower plate, closest to the Earth's surface represents the bulk of the atmosphere and it is optically thick in the major absorption bands of its gaseous components. The upper plate is the optically thin part of the atmosphere from which infrared radiation from those bands is lost to free space. The lower, denser atmospheric level has an average temperature of $T(c)$, an average absorptivity of $a(c)$, and an average emissivity of $e(c)$. The upper, less dense portion of the atmosphere is optically thinner with an average temperature of $T(h)$, and average absorptivity of $a(h)$, and an average emissivity of $e(h)$.

If the hotter Earth's surface were facing a complete void at 0 K, the flux of radiant energy that it would emit to that void would be:

$$e(h) \sigma T(h)^4 \quad (1)$$

where σ is the Stefan-Boltzmann constant. Similarly, if the colder atmosphere were facing such a void, the flux of radiant energy that it would emit to that void would on the average be:

$$e(c) \sigma T(c)^4 \quad (2)$$

When they are facing each other as they are on the Earth, the colder atmosphere absorbs a flux of radiation emitted from the hotter Earth surface of:

$$a(c) e(h) \sigma T(h)^4 \quad (3)$$

The warmer earth then absorbs a flux of radiation emitted from the colder atmosphere of:

$$a(h) e(c) \sigma T(c)^4 \quad (4)$$

.... the net transfer of the radiant energy flux between the two surfaces is:

$$(3) - (4): I(\text{net}) = a(c) e(h) \sigma T(h)^4 - a(h) e(c) \sigma T(c)^4, \sigma = 5.67$$

$$I(\text{net}) = 5.67 [a(c) e(h) T(h)^4 - a(h) e(c) T(c)^4] \quad (5)$$

This general case is true without invoking the special case of Kirchhoff's Law, $e(h) = a(h)$ or $e(c) = a(c)$, which is only valid when the radiator has no non-radiant energy transfer mechanisms affecting it, like thermal and convective heat transfer, condensing, melting, electrical, magnetic, chemical, mechanical and gravitational energy transformations.

Example 1. General. Neither radiator follows Kirchhoff's Law.

$$T(h) = 275/100$$

$$T(c) = 274/100$$

$$a(c) = 0.4$$

$$a(h) = 0.6$$

$$e(h) = 0.3$$

$$e(c) = 0.5$$

$$I(\text{net}) = \sigma [0.4 * 0.3 * 2.75^4 - 0.6 * 0.5 * 2.74^4]$$

$$I(\text{net}) = \sigma [0.12 * 2.75^4 - 0.3 * 2.74^4]$$

$$I(\text{net}) = \sigma [0.12 * 57.191 - 0.3 * 56.364]$$

$$I(\text{net}) = \sigma [6.863 - 16.909]$$

$$I(\text{net}) = 5.67 [- 10.0462] = - 56.962 \text{ w/m}^2$$

This means radiant energy flows from 274K radiator to 275K radiator.

Example 2. Only cold radiator obeys Kirchhoff's Law, hot does not.

$$T(h) = 275/100$$

$$T(c) = 274/100$$

$$a(c) = 0.5$$

$$a(h) = 0.6$$

$$e(h) = 0.3$$

$$e(c) = 0.5$$

$$I(\text{net}) = \sigma [0.5 * 0.3 * 2.75^4 - 0.6 * 0.5 * 2.74^4]$$

$$I(\text{net}) = \sigma [0.15 * 2.75^4 - 0.3 * 2.74^4]$$

$$I(\text{net}) = \sigma [0.15 * 57.191 - 0.3 * 56.364]$$

$$I(\text{net}) = \sigma [8.579 - 16.909]$$

$$I(\text{net}) = 5.67 [- 8.331] = - 47.234 \text{ w/m}^2$$

This means radiant energy flows from 274K radiator to 275K radiator.

Example 3. Both radiators obey Kirchhoff's Law

$$T(h) = 275/100$$

$$T(c) = 274/100$$

$$a(c) = 0.5$$

$$a(h) = 0.6$$

$$e(h) = 0.6$$

$$e(c) = 0.5$$

$$I(\text{net}) = \sigma [0.5 * 0.6 * 2.75^4 - 0.6 * 0.5 * 2.74^4]$$

$$I(\text{net}) = \sigma [0.3 * 2.75^4 - 0.3 * 2.74^4]$$

$$I(\text{net}) = \sigma [0.3 * 57.191 - 0.3 * 56.364]$$

$$I(\text{net}) = \sigma [17.157 - 16.909]$$

$$I(\text{net}) = 5.67 [- 8.331] = + 0.248 \text{ w/m}^2$$

This means radiant energy flows from 275K radiator to 274K radiator.

In this special case where Kirchhoff's Law applies, the rate law simplifies to the one commonly used by GHGT theorists.

$$I(\text{net}) = \sigma a(c) e(h) [T(h)^4 - T(c)^4] = \sigma a(h) e(c) [T(h)^4 - T(c)^4] \quad (5)$$

So the Hertzberg general rate law disproves the notion radiant energy transfer only flows from the hot radiator to the cold one. That is only true if one radiator is sufficiently hotter than the other or both radiators obey Kirchhoff's Law, emissivity = absorptivity. That is not easy to guarantee. The Earth's atmosphere has several energy transfer mechanisms within it and hence does not obey Kirchhoff's Law.

References

1. Hertzberg, M, "The Night Time Radiative Transport Between the Earth's Surface, Its Atmosphere, and Free Space", *Energy & Environment*, V23, n5, 2012, p 821. <http://climaterealist.com/attachments/ftp/05-Hertzberg.pdf>
2. Latour, PR, "Radiation Physics Laws Give the Effect of CO2 on Earth's Temperatures – A Primer", *Principia Scientific International*, February 25, 2017. <http://principia-scientific.org/radiation-physics-laws-give-effect-co2-earths-temperatures-primer/>